Direktorat Riset dan Pengabdian Masyarakat Direktorat Jenderal Riset dan Pengembangan
Kementerian Riset, Teknologi, dan Pendidikan Tinggi
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LAPORAN AKHIR PENELITIAN TAHUN TUNGGAL

ID Proposal: 52513523-4882-438f-bddb-378eb4b54a64
Laporan Akhir Penelitian: tahun ke-2 dari 2 tahun

1. IDENTITAS PENELITIAN

A. JUDUL PENELITIAN

MODEL PREDIKSI LOKASI WAKTU MENGGUNAKAN MODEL GSTAR MULTIVARIAT GARCH, SUDI KASUS: KECEPATAN ANGIN DAN TINGGI GELOMBANG LAUT DI PERAIRAN LAUT MANADO DAN BITUNG

B. BIDANG, TEMA, TOPIK, DAN RUMPUN BIDANG ILMU

Bidang Fokus RIRN / Bidang Unggulan Perguruan Tinggi	Tema	Topik (jika ada)	Rumpun Bidang Ilmu
Manajemen Penanggulangan Kebencanaan dan Lingkungan	-	Penguatan mitigasi dan adaptasi terhadap perubahan iklim global	Statistik

C. KATEGORI, SKEMA, SBK, TARGET TKT DAN LAMA PENELITIAN

Kategori (Kompetitif Nasional/ Desentralisasi/ Penugasan)	Skema Penelitian	Strata (Dasar/ Terapan/ Pengembangan)	SBK (Dasar, Terapan, Pengembangan)	Target Akhir TKT	Lama Penelitian (Tahun)
Penelitian Desentralisasi	Penelitian Dasar Unggulan Perguruan Tinggi	SBK Riset Dasar	SBK Riset Dasar	2	2

2. IDENTITAS PENGUSUL

Nama, Peran	Perguruan Tinggi/ Institusi	Program Studi/ Bagian	Bidang Tugas	ID Sinta	H-Index
NELSON NAINGGOLAN Ketua Pengusul	Universitas Sam Ratulangi	Matematika		5975252	0
TOHAP MANURUNG S.Si.,M.Si Anggota Pengusul 2	Universitas Sam Ratulangi	Matematika	Pengolahan data, analisis prediksi (forecasting)	6197760	0
Dr.Ir. HANNY	Universitas	Matematika	Pemodelan ,	6587191	0

ANDREA HUIBERT KOMALIG M.Si. Anggota Pengusul 1	Sam Ratulangi		analisis data multivariat		

3. MITRA KERJASAMA PENELITIAN (JIKA ADA)

Pelaksanaan penelitian dapat melibatkan mitra kerjasama, yaitu mitra kerjasama dalam melaksanakan penelitian, mitra sebagai calon pengguna hasil penelitian, atau mitra investor

Mitra	Nama Mitra

4. LUARAN DAN TARGET CAPAIAN

Luaran Wajib

Tahun Luaran	Jenis Luaran	Status target capaian (*accepted, published, terdaftar atau granted, atau status lainnya*)	Keterangan (*url dan nama jurnal, penerbit, url paten, keterangan sejenis lainnya*)
2	Publikasi Ilmiah Jurnal Internasional	accepted/published	International journal of science and research (IJSR) : (https://www.ijsr.net/)

Luaran Tambahan

Tahun Luaran	Jenis Luaran	Status target capaian (*accepted, published, terdaftar atau granted, atau status lainnya*)	Keterangan (*url dan nama jurnal, penerbit, url paten, keterangan sejenis lainnya*)
2	Prosiding dalam pertemuan ilmiah Nasional	sudah terbit/sudah dilaksanakan	Seminar nasional sains dan terapan (sinta)

5. ANGGARAN

Rencana anggaran biaya penelitian mengacu pada PMK yang berlaku dengan besaran minimum dan maksimum sebagaimana diatur pada buku Panduan Penelitian dan Pengabdian kepada Masyarakat Edisi 12.

Total RAB 2 Tahun Rp. 58,200,000

Tahun 1 Total Rp. 0

Tahun 2 Total Rp. 58,200,000

Jenis Pembelanjaan	Item	Satuan	Vol.	Biaya Satuan	Total
Analisis Data	HR Sekretariat/Administrasi Peneliti	OB	1	2,000,000	2,000,000
Analisis Data	HR Pengolah Data	P (penelitian)	1	2,500,000	2,500,000
Analisis Data	Transport Lokal	OK (kali)	5	1,320,000	6,600,000
Bahan	ATK	Paket	1	9,200,000	9,200,000
Bahan	Bahan Penelitian (Habis Pakai)	Unit	5	1,500,000	7,500,000
Pelaporan, Luaran Wajib, dan Luaran Tambahan	HR Sekretariat/Administrasi Peneliti	OB	1	2,000,000	2,000,000
Pelaporan, Luaran Wajib,	Biaya seminar nasional	Paket	1	400,000	400,000

Jenis Pembelanjaan	Item	Satuan	Vol.	Biaya Satuan	Total
dan Luaran Tambahan					
Pelaporan, Luaran Wajib, dan Luaran Tambahan	Biaya seminar internasional	Paket	1	6,500,000	6,500,000
Pelaporan, Luaran Wajib, dan Luaran Tambahan	Publikasi artikel di Jurnal Internasional	Paket	1	8,000,000	8,000,000
Pengumpulan Data	FGD persiapan penelitian	Paket	1	1,000,000	1,000,000
Pengumpulan Data	HR Sekretariat/Administrasi Peneliti	OB	1	1,000,000	1,000,000
Pengumpulan Data	HR Pembantu Peneliti	OJ	2	3,000,000	6,000,000
Pengumpulan Data	Transport	OK (kali)	5	500,000	2,500,000
Pengumpulan Data	Biaya konsumsi	OH	5	200,000	1,000,000
Sewa Peralatan	Peralatan penelitian	Unit	1	2,000,000	2,000,000

6. HASIL PENELITIAN

A. RINGKASAN: Tuliskan secara ringkas latar belakang penelitian, tujuan dan tahapan metode penelitian, luaran yang ditargetkan, serta uraian TKT penelitian.

Pada umumnya fenomena dilapangan terutama yang berkaitan dengan data riil merupakan time series dengan pola heteroskedastik (Lo, 2003), (Zivot, 2006). Penelitian ini mengembangkan model Generalisasi space time autoregresi (GSTAR) dengan galat berbentuk multivraiat GARCH (Generalized Autoregressive Conditional Heterroscedasticity). Hasil yang diperoleh adalah model modifikasi dari model GSTAR dengan galat multivariat GARCH. Bentuk multivariat GARCH yang akan dikerjakan meliputi tiga bentuk yaitu bentuk vector Half (VECH), Parameterisasi Definit Positif oleh BEKK (Baba, Engle, Kraft dan Kroner) dan Korelasi Konstan (CC). Pada tahun pertama ini diuraikan rumus-rumus model multivariat GARCH dan rumus-rumusnya dibuktikan. Juga telah dihasilkan persamaan matematika model GSTAR(1,1) untuk data kecepatan angin dan tinggi gelombang laut di perairan laut Manado dan Bitung. Kemudian disusun program-program menggunakan Software S-Plus8 atau program R untuk penaksiran parameter dan analisis-analisis statistika. Luaran dari peneltian ini adalah makalah yang sudah dipublikasikan pada : International Journal of Recent Technology and Engineering (IJRTE), Volume-8 Issue-2S7

https://www.ijrte.org/download/volume-8-issue-2S7/, juga makalah yang sudah dipresentasikan pada konferensi internasional The 3rd ICOR pada tanggal 20 September 2018 dan The 4th ICOR 19 September 2019 di Manado, yang mana papernya akan dipublikasikan pada Jurnal Internasional.

B. KATA KUNCI: Tuliskan maksimal 5 kata kunci.

Model GSTAR; GARCH; kecepatan angin; tinggi gelombang

Pengisian poin C sampai dengan poin H mengikuti template berikut dan tidak dibatasi jumlah kata atau halaman namun disarankan seringkas mungkin. Dilarang menghapus/memodifikasi template ataupun menghapus penjelasan di setiap poin.

C. HASIL PELAKSANAAN PENELITIAN: Tuliskan secara ringkas hasil pelaksanaan penelitian yang telah dicapai sesuai tahun pelaksanaan penelitian. Penyajian dapat berupa data, hasil analisis, dan capaian luaran (wajib dan atau tambahan). Seluruh hasil atau capaian yang dilaporkan harus berkaitan dengan tahapan pelaksanaan penelitian sebagaimana direncanakan pada proposal. Penyajian data dapat berupa gambar, tabel, grafik, dan sejenisnya, serta analisis didukung dengan sumber pustaka primer yang relevan dan terkini.

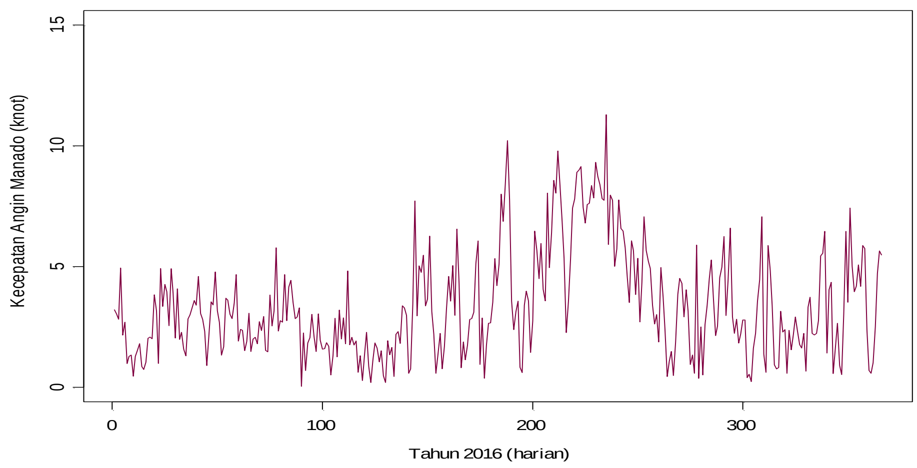
Pengisian poin C sampai dengan poin H mengikuti template berikut dan tidak dibatasi jumlah kata atau halaman namun disarankan seringkas mungkin. Dilarang menghapus/memodifikasi template ataupun menghapus penjelasan di setiap poin.

C. **HASIL PELAKSANAAN PENELITIAN:** Tuliskan secara ringkas hasil pelaksanaan penelitian yang telah dicapai sesuai tahun pelaksanaan penelitian. Penyajian dapat berupa data, hasil analisis, dan capaian luaran (wajib dan atau tambahan). Seluruh hasil atau capaian yang dilaporkan harus berkaitan dengan tahapan pelaksanaan penelitian sebagaimana direncanakan pada proposal. Penyajian data dapat berupa gambar, tabel, grafik, dan sejenisnya, serta analisis didukung dengan sumber pustaka primer yang relevan dan terkini.

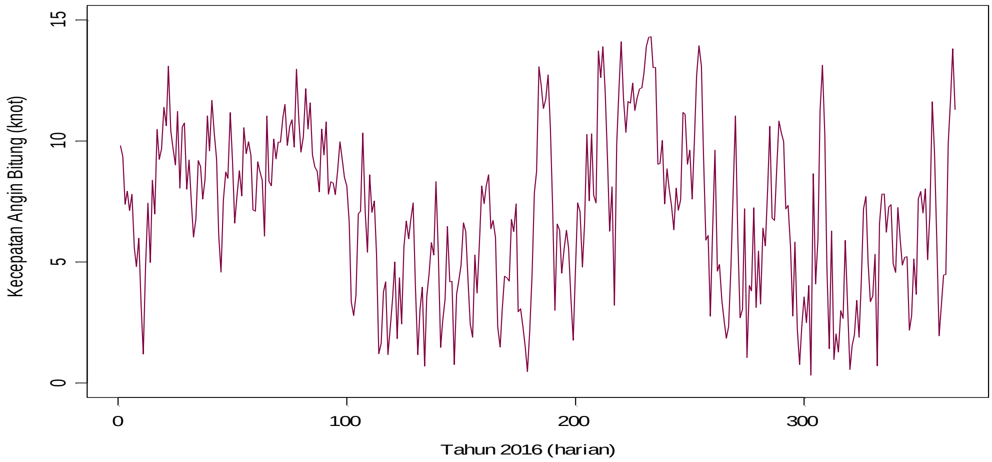
1. Data Kecepatan Angin dan Tinggi Gelombang Laut.

Data kecepatan angin dan tinggi gelombang laut yang diambil adalah data harian tahun 2012 sampai dengan tahun 2016. Data ini merupakan data sekunder yang diperoleh dari kantor BMKG Bitung. Data yang diambil merupakan data di dua lokasi yaitu perairan laut manado dan Bitung. Lokasi data kecepatan angin dan tinggi gelombang untuk perairan laut Manado adalah pada 1,6 LU dan 124,5 BT, sedangkan untuk perairan laut Bitung adalah pada 1,2 LU dan 125,3 BT.

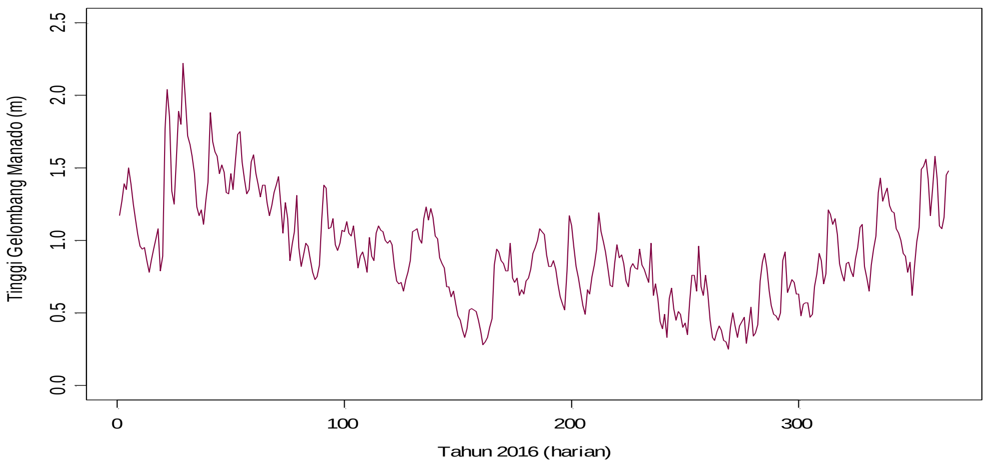
Berikut ini diberikan grafik data harian tahun 2016, kecepatan angin dan tinggi gelombang di perairan laut Manado dan Bitung. Grafik ini dibuat dengan terlebih dahulu menyusun program pembuatan grafik menggunakan S-PLUS8 [1] [2].



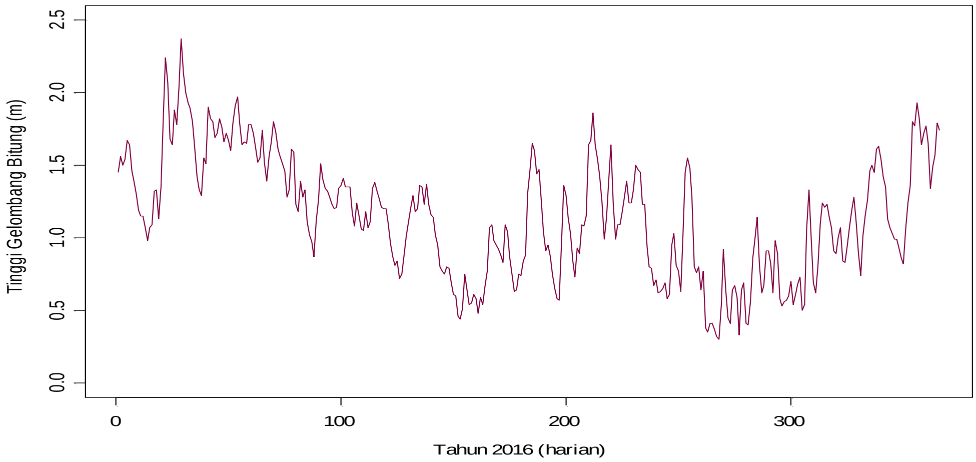
Gambar 1. Grafik Kecepatan Angin (knot) di Perairan Laut Manado Tahun 2016



Gambar 2. Grafik Kecepatan Angin (knot) di Perairan Laut Bitung Tahun 2016



Gambar 3. Grafik Tinggi Gelombang (m) di Perairan Laut Manado Tahun 2016



Gambar 4. Grafik Tinggi Gelombang (m) di Perairan Laut Bitung Tahun 2016

Pada Gambar 1 dan Gambar 2 menunjukkan adanya korelasi atau kemiripan pola kecepatan angin diperairan laut Manado dan Bitung. Demikian juga untuk Gambar 3 dan Gambar 4 menunjukkan adanya pola tinggi gelombang yang mirip. Namun hal ini akan lebih jelas lagi jika dinyatakan dalam model GSTAR.

2. Model GSTAR(1,1) Kecepatan Angin dan Tinggi Gelombang Laut

Ruchjana [3] memperluas model STAR menjadi model GSTAR yaitu :

$$\mathbf{Z}_t = \sum_{k=1}^{p} \sum_{l=0}^{\lambda_k} \mathbf{\Phi}_{kl} \mathbf{W}^{(l)} \mathbf{Z}_{t-k} + \boldsymbol{\varepsilon}_t$$

dimana : $\mathbf{\Phi}_{kl}$ disebut parameter autoregressive pada lag waktu k dan lag spasial l. Sebagai contoh, model GSTAR (1;1) untuk N-lokasi adalah berbentuk :

$$\begin{pmatrix} Z_{1,t} \\ \vdots \\ Z_{N,t} \end{pmatrix} = \begin{pmatrix} \phi_{10}^{(1)} & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & \phi_{10}^{(N)} \end{pmatrix} \begin{pmatrix} Z_{1,t-1} \\ \vdots \\ Z_{N,t-1} \end{pmatrix} + \begin{pmatrix} \phi_{11}^{(1)} & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & \phi_{11}^{(N)} \end{pmatrix} \begin{pmatrix} w_{11}^{(1)} & \cdots & w_{1N}^{(1)} \\ \vdots & & \vdots \\ w_{N1}^{(1)} & \cdots & w_{NN}^{(1)} \end{pmatrix} \begin{pmatrix} Z_{1,t-1} \\ \vdots \\ Z_{N,t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{N,t} \end{pmatrix}$$

dan model GSTAR(1,1) untuk 2 lokasi dapat dituliskan sebagai berikut:

$$\begin{pmatrix} Z_1(t) \\ Z_2(t) \end{pmatrix} = \begin{pmatrix} \phi_{10}^{(1)} & 0 \\ 0 & \phi_{10}^{(2)} \end{pmatrix} \begin{pmatrix} Z_1(t-1) \\ Z_2(t-1) \end{pmatrix} + \begin{pmatrix} \phi_{11}^{(1)} & 0 \\ 0 & \phi_{11}^{(2)} \end{pmatrix} \begin{pmatrix} w_{11}^{(1)} & w_{12}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} \end{pmatrix} \begin{pmatrix} Z_1(t-1) \\ Z_2(t-1) \end{pmatrix} + \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} .$$

Misalkan Y1 dan Y2 berturut-turut adalah kecepatan angin di perairan laut Manado dan Bitung. Grafik tinggi data kecepatan angin dapat dilihat pada Gambar 1 dan Gambar 2. Pada pemodelan GSTAR ini matriks bobot yang digunakan adalah bobot seragam, hal ini dikarenakan hanya ada dua lokasi. Maka penerapan model GSTAR(1,1) dua lokasi untuk kecepatan angin di perairan laut Manado dan Bitung adalah

$$Y_1(t) = 0{,}5942\, Y_1(t-1) + 0{,}1721\, Y_2(t-1)$$
$$Y_2(t) = 0{,}7638\, Y_2(t-1) - 0{,}0097\, Y_1(t-1)$$

Misalkan Z1 dan Z2 berturut-turut adalah tinggi gelombang laut di perairan laut Manado dan Bitung. Grafik tinggi data tinggi gelombang laut dapat dilihat pada Gambar 3 dan Gambar 4. Juga pada pemodelan GSTAR ini matriks bobot yang digunakan adalah bobot seragam, hal ini dikarenakan hanya ada dua lokasi. Maka penerapan model GSTAR(1,1) dua lokasi untuk tinggi gelombang laut di perairan laut Manado dan Bitung adalah

$$Z_1(t) = 0{,}7418\, Z_1(t-1) + 0{,}1721\, Z_2(t-1)$$
$$Z_2(t) = 0{,}8998\, Z_2(t-1) + 0{,}0328\, Z_1(t-1)$$

Kecepatan angin dan tinggi gelombang laut di perairan laut Manado dan Bitung dipengaruhi oleh kecepatan angin dan tinggi gelombang laut satu waktu sebelumnya pada lokasi yang sama dan dari lokasi sekitarnya.

3. Model Multivariat GARCH

Secara umum proses ARCH(q) didefinisikan sebagai berikut. Anggap bahwa $Y_1, Y_2, \cdots\cdots, Y_T$ adalah observasi time series dan misalkan Ft himpunan semua Yt hingga waktu t, termasuk Yt untuk $t \le 0$. Process {Yt} dikatakan suatu proses Autoregressive Conditional Heteroscedastic orde q, ARCH(q), jika :

$Y_t | F_{t-1}$ ~ N(0, ht),

dan $h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \cdots\cdots + \alpha_q Y_{t-q}^2$

dimana : q > 0, α0 > 0, dan αi ≥ 0, untuk i = 1, 2, ……., q,[4].

Selanjutnya jika pada proses ARCH(q) dimasukkan lag dari σt2 maka diperoleh model GARCH(p,q), yaitu :

$$h_t = \alpha_0 + \beta_1 h_1 + \cdots + \beta_p h_{t-p} + \alpha_1 Y_{t-1}^2 + \cdots\cdots + \alpha_q Y_{t-q}^2$$

dimana p menyatakan lag pada σt2 dan q menyatakan lag pada Yt 2 [5]. Secara khusus untuk p =1 dan q = 1 diperoleh model GARCH(1,1) yaitu :

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_1 h_{t-1}$$

Dalam hal ini :

$E(Y_t | F_{t-1}) = 0,$ dan $Var(Y_t | F_{t-1}) = E(Y_t^{\ 2} | F_{t-1}) = h_t$

Apabila pada (10) dibagi dengan akar kuadrat dari variansi bersyarat dari Yt , maka diperoleh :

$\frac{Y_t}{\sqrt{h_t}} | F_{t-1}$ ~ N(0, 1)

Oleh karena itu barisan Z1 , …… , ZT , yang didefinisikan dengan

$Z_t = \frac{Y_t}{\sqrt{h_t}}$ atau $Y_t = Z_t \sqrt{h_t}$

merupakan barisan i.i.d. N(0,1). Oleh karena itu kita dapat mengkonstruksi solusi stasioner dari (14) dimulai dengan mengkonstruksi barisan variabel acak {Zt}, i.i.d. N(0,1).

Perluasan dari model univariat GARCH menjadi model m-variat memerlukan matiks varians-kovariansi bersyarat dari variabel acak εt , dengan rataan nol dan m-dimensi, bergantung pada elemen dari himpunan informasi. Misalkan {Zt} adalah vektor variabel acak i.i.d. berukuran (mx1) dengan karakteristik sebagai berikut:

E(Zt) = 0

E(Zt Zt ′) = Im

Zt ~ G(0,Im)

dengan G merupakan fungsi padat kontinu. Misalkan pula { εt }adalah barisan vektor acak berukuran (mx1) yang berbentuk

$$\varepsilon_t = Z_t \sqrt{H_t}$$

dengan

Et-1(εt) = 0 dan Et-1(εt εt ′) = Ht

dan Ht adalah matriks kovariansi vektor { εt }, definit positif berukuran mxm dan terukur terhadap himpunan informasi Ft-1, yaitu σ-field yang dibangun oleh informasi lampau: { εt-1, εt-2, ...}.

Parameterisasi dari Ht sebagai sebuah multivariat GARCH, yang berarti sebagai fungsi dari himpunan informasi dari Ft-1, maka elemen dari Ht adalah bergantung pada lag-q dari kuadrat εt dan cross-products εt . Dengan demikian elemen dari matriks covarians tersebut mengikuti sebuah vektor proses ARMA dalam kuadrat dan cross-products dari suku gangguan (error). Parameterisasi dari Ht sebagai sebuah multivariat ARCH (GARCH) diberikan dalam tiga bentuk, yaitu model Vech, model BEKK dan model Korelasi Konstan, [6][7].

3.1 Representasi Vech

Vech adalah operator vector-half (setengah vektor) yaitu menyususn (menumpuk) elemen-elemen segitiga bawah dari matrik mxm menjadi vektor berukuran (m(m+1)/2)x1. Representasi vech sering dinamakan dengan pamrameterisasi penuh (full parameterisation). Berhubung karena matriks covarians Ht adalah matriks symetri, maka vech(Ht) mengandung elemen-elemen di Ht secara tunggal. Dengan demikian maka perluasan model multivariat GARCH(p,q) dalam representasi vech, dapat dituliskan sebagai:

$$vech\ (H_t) = W + \sum_{i=1}^{q} A_i\ vech(\varepsilon_{t-i}\ \varepsilon_{t-i}^{'}) + \sum_{j=1}^{p} B_j vech(H_{t-j})$$

dimana : W adalah vektor berukuran (m(m+1)/2) x 1, sedangkan Ai dan Bj adalah matriks berukuran (n(n+1)/2) x (n(n+1)/2).

Jumlah parameter dalam formulasi umum representasi vech adalah sebanyak {m(m+1)/2 + (p+q) (m(m+1)/2)2}. Sebagai contoh, misalkan m =2, dan p = q =1, maka jumlah parameter adalah 21 buah. Bentuk model vech(Ht) untuk contoh ini adalah sebagai berikut:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}.$$

Dalam hal ini, elemen ke (i,j) dalam Ht bergantung pada elemen ke (i,j) yang bersesuaian dalam εtεt’ dan Ht-1 . Untuk menjamin bahwa matriks Ht definit positip, syarat-syarat yang diperlukan adalah:

w1>0 , w3>0 , w1w3-w22 >0,

a11≥0, a13≥0, a31≥0, a33≥0, a11a33-a22a22≥0, a11a13- (1/4) a12a12 ≥0, a11a31-a21a21≥0,

a31a33- (1/4) a32a32 ≥0, a13a33-a23a23≥0.

Untuk mengurangi jumlah parameter yang besar maka dilakukan penyederhanaan pada representasi vech. Salah satu cara adalah dengan memilih matriks A dan B dalam bentuk diagonal. Ini dinamakan model vech diagonal. Penyederhanaan ini mengurangi jumlah parameter dimana banyak parameter menjadi (m(m+1)/2)(1+p+q). Misalkan untuk m = 2, dan p = q = 1, maka model vech diagonal dapat dituliskan sebagai:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}$$

Dengan mengalikan matriks di atas diperoleh:

$$h_{11,t} = w_1 + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}$$

$$h_{21,t} = w_2 + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{22}h_{21,t-1}$$

$$h_{22,t} = w_3 + a_{33}\varepsilon_{2,t-1}^2 + b_{33}h_{22,t-1}$$

Dalam hal ini jumlah parameternya berkurang dari 21 buah desederhanakan menjadi hanya 9 buah. Dengan pengurangan parameter ini, maka syarat-syarat agar matriks Ht definit positip adalah:

w1>0 , w3>0 , w1w3-w22 >0,

a11≥0, a33≥0, a11a33-a22a22≥0.

3.2 Parameterisasi Definit Positip

Parameterisasi definit positip ini dikenal juga denga nama BEKK (Baba, Engle, Kraft dan Kroner). Model parameterisasi dari BEKK adalah berbentuk:

$$H_t = CC' + \sum_{k=1}^{K} \sum_{i=1}^{q} A_{ik}\varepsilon_{t-i}\varepsilon_{t-i}' A_{ik}' + \sum_{k=1}^{K}\sum_{i-1}^{p} B_{ik} H_{t-i} B_{ik}'$$

dimana C adalah matriks segitiga bawah, sehingga CC′ adalah matriks parameter symetri (mxm), sedangkan Aik dan Bik adalah matriks parameter sebarang (mxm). Model ini menjamin bahwa matriks Ht adalah matriks definit positip. Sebagai contoh, ambil K = p = q = 1 dan N = 2. Maka model BEKK diatas adalah berbentuk:

$$\begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} = \begin{pmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} c_{11} & c_{21} \\ 0 & c_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$$
$$+ \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{pmatrix} \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix}$$

Maka dengan mengalikan matriks tersebut diperoleh:

$$h_{11,t} = c_{11}^2 + (a_{11}\varepsilon_{1,t-1} + a_{12}\varepsilon_{2,t-1})^2 + (b_{11}h_{11,t-1} + b_{12}h_{22,t-1})^2$$

$$h_{22,t} = c_{21}^2 c_{22}^2 + (a_{21}\varepsilon_{1,t-1} + a_{22}\varepsilon_{2,t-1})^2 + (b_{21}h_{11,t-1} + b_{22}h_{22,t-1})^2$$

$$h_{21,t} = h_{12,t} = c_{11}c_{21} + a_{11}a_{21}\varepsilon^2_{1,t-1} + (a_{11}a_{22} + a_{12}a_{21})\varepsilon_{1,t-1}\varepsilon_{2,t-1}$$
$$+ a_{12}a_{22}\varepsilon^2_{2,t-1} + b_{11}b_{21}h_{11,t-1} + (b_{11}b_{22} + b_{12}b_{21})h_{21,t-1}$$
$$+ b_{12}b_{22}h_{22,t-1}$$

3.3 Model Korelasi Konstan

Dalam model korelasi konstan, kovarians kondisional adalah diparameterisasi proporsional terhadap hasil kali deviasi standar kondisional yang bersesuaian. Dalam hal ini, diasumsikan bahwa matriks korelasi εt , kondisional, adalah konstan. Asumsi ini menyederhanakan beban perhitungan dalam estimasi dan juga syarat agar Ht definit positip untuk setiap t mudah diperoleh. Jadi asumsi dalam model ini adalah:

Et-1[εt] = 0

Et-1[εt εt′] = Ht

$$\{H_t\}_{ii} = h_{ii,t}$$

$$\{H_t\}_{ij} = h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}} \quad jika \;\; i \neq j$$

Misalkan Dt menyatakan matriks diagonal mxm dengan unsur diagonalnya adalah varians kondisional, yaitu $\{D_t\}_{ii} = h_{ii,t}$. Misalkan Γt menyatakan matriks korelasi konstan dimana elemen ke-ij adalah:

$$\{\Gamma_t\}_{ij} = \{H_t\}_{ij}\left(\{H_t\}_{ii}\{H_t\}_{jj}\right)^{-1/2} \qquad i, j = 1,2,\ldots,m$$

Dengan asumsi pada model Γt = Γ, maka

Ht = Dt1/2 Γ Dt1/2

$$H_t = diag(\sqrt{h_{11,t}},\cdots,\sqrt{h_{mm,t}})\;\Gamma_t\;diag(\sqrt{h_{11,t}},\cdots,\sqrt{h_{mm.t}})$$

Sebagai contoh, untuk N = 2, p = q = 1. Maka

$$\mathbf{H}_t = \begin{pmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{pmatrix}\begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix}\begin{pmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{pmatrix}$$

Dalam hal ini,

$\mathbf{D}_t = \begin{pmatrix} h_{11,t} & 0 \\ 0 & h_{22,t} \end{pmatrix}$, dan $\mathbf{\Gamma}_t = \begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix}$,

sehingga: $\mathbf{H}_t = \begin{pmatrix} h_{11,t} & \rho\sqrt{h_{11,t}h_{22,t}} \\ \rho\sqrt{h_{11,t}h_{22,t}} & h_{22,t} \end{pmatrix}$.

dimana: ρ = ρ12 = ρ21 adalah koefisien korelasi antara ε1,t dan ε2,t dan memenuhi sifat korelasi yaitu │ρ│< 1, sedangkan h11,t dan h22,t merupakan varians untuk proses ARCH(p,q) standar. Misalkan untuk p = q = 1, maka

$$h_{11,t} = \alpha_{10} + \alpha_{11}\varepsilon^2_{1,t-1} + \beta_{11}h_{11,t-1}$$

$$h_{22,t} = \alpha_{20} + \alpha_{21}\varepsilon^2_{2,t-1} + \beta_{21}h_{22,t-1}$$

$$h_{12,t} = h_{21,t} = \rho\sqrt{\alpha_{10} + \alpha_{11}\varepsilon_{1,t-1}^2 + \beta_{11}h_{11,t-1}}\sqrt{\alpha_{20} + \alpha_{21}\varepsilon_{2,t-1}^2 + \beta_{21}h_{22,t-1}}$$

Jumlah parameter yang akan ditaksir adalah 6 buah. Untuk memenuhi syarat dimana matriks Ht definit positip maka dalam model korelasi konstan ini diperlukan syarat-syarat:

αi0 > 0, αij ≥ 0 dan βik ≥ 0,

i = 1, ..., m, j = 1, ..., q, dan k = 1, ..., p.

Apabila varians kondisional sepanjang diagonal matriks D semua positip, maka matriks korelasi Γ adalah definit positip dan hal ini mengakibatkan barisan matriks kovarians kondisional {Ht} dapat dipastikan definit positip hampir pasti untuk semua t. Selanjutnya invers dari Ht adalah:

Ht-1 = Dt-1/2 Γ-1Dt-1/2 .

4. PENAKSIRAN PARAMETER

Misalkan {yt} , t = 1, 2, ..., T, merupakan realisasi dari vektor proses stokastik, dengan rataan vektor μt = 0 dan matriks kovarians kondisional adalah Ht(θ), dimana θ adalah vektor parameter. Kepadatan multivariat normal [8] dari vektor proses tersebut adalah:

$$f(\mathbf{y}_t |\ F_{k-1}) = \frac{1}{\sqrt{(2\pi)^N}\left|\mathbf{H}_t\right|^{1/2}} \exp\left\{\frac{-1}{2}\mathbf{y}_t'\ \mathbf{H}_t^{-1}\mathbf{y}_t\right\}$$, t = 1, 2, …., T.

Penaksiran parameter model multivariat ARCH (GARCH) menggunakan metode maksimum likelihod [7]. Fungsi log-likelihood untuk kepadatan gabungan multivariat normal [9] adalah:

$$L(\theta) = \frac{-1}{2}\sum_{t=1}^{T}\left|\mathbf{H}_t\right| - \frac{1}{2}\sum_{t=1}^{T}\mathbf{y}_t'\mathbf{H}_t^{-1}\mathbf{y}_t$$

Selanjutnya kita dapat menaksir θ dengan menyelesaikan turunan dari fungsi log likelihood $\frac{\partial L}{\partial \boldsymbol{\theta}} = 0$.

Penaksiran parameter untuk model vech diagonal adalah sebagai berikut:

$$H_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix}$$

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}$$

Parameter θ = (w1, w2, w3, a11, a22, a33, b11, b22, b33)′ , yaitu ada 9 parameter yang akan ditaksir.

Penyelesaian $\frac{\partial L}{\partial \boldsymbol{\theta}} = 0$ adalah menyelesaian secara simultan 9 persamaan, yaitu: $\frac{\partial L}{\partial w_1} = 0$, $\frac{\partial L}{\partial w_2} = 0$, $\frac{\partial L}{\partial w_3} = 0$,

$\frac{\partial L}{\partial a_{11}} = 0$, $\frac{\partial L}{\partial a_{22}} = 0$, $\frac{\partial L}{\partial a_{33}} = 0$,

$\frac{\partial L}{\partial b_{11}} = 0$, $\frac{\partial L}{\partial b_{22}} = 0$ dan $\frac{\partial L}{\partial b_{33}} = 0$.

Jika dibandingkan dengan model korelasi konstan, maka penaksiran parameternya adalah sebagai berikut:

$$\mathbf{H}_t = \begin{pmatrix} h_{11,t} & \rho\sqrt{h_{11,t}h_{22,t}} \\ \rho\sqrt{h_{11,t}h_{22,t}} & h_{22,t} \end{pmatrix}$$

$$h_{11,t} = \alpha_{10} + \alpha_{11}\varepsilon_{1,t-1}^2 + \beta_{11}h_{11,t-1}$$

$$h_{22,t} = \alpha_{20} + \alpha_{21}\varepsilon_{2,t-1}^2 + \beta_{21}h_{22,t-1}$$

$$h_{12,t} = h_{21,t} = \rho\sqrt{\alpha_{10} + \alpha_{11}\varepsilon_{1,t-1}^2 + \beta_{11}h_{11,t-1}}\sqrt{\alpha_{20} + \alpha_{21}\varepsilon_{2,t-1}^2 + \beta_{21}h_{22,t-1}}$$

Parameter θ = (α10, α11, β11, α20, α21, β21)′ . Penyelesaian $\frac{\partial L}{\partial \boldsymbol{\theta}} = 0$ adalah menyelesaian secara simultan 6 persamaan, yaitu:

$$\frac{\partial L}{\partial \alpha_{10}} = 0,\ \frac{\partial L}{\partial \alpha_{11}} = 0,\ \frac{\partial L}{\partial \alpha_{20}} = 0,\ \frac{\partial L}{\partial \alpha_{21}} = 0,\ \frac{\partial L}{\partial \beta_{11}} = 0,\ \frac{\partial L}{\partial \beta_{21}} = 0 \quad .$$

5. Model GSTAR- Multivariat GARCH

Misalkan Z(t) = (Z1(t), ... , ZN(t))' adalah vektor time series dengan rataan vektor nol, E[Z(t)] = 0, dan ε(t) = (ε1(t), ... , εN(t))′ , merupakan vektor acak i.i.d. dengan mean nol dan variansi konstan. Model GSTAR(1;1)-GARCH(1) dinyatakan dengan

$$\mathbf{Z}(t) = \Phi_{10}\mathbf{Z}(t-1) + \Phi_{11}\mathbf{W}\ \mathbf{Z}(t-1) + \boldsymbol{\varepsilon}(t) \qquad (1)$$

$$\boldsymbol{\varepsilon}(t) = \mathbf{D}_t\boldsymbol{\eta}_t$$

$$(\varepsilon_t | F_{t-1}) \sim N(\mathbf{0}, \boldsymbol{\Sigma}_t)$$

dengan ηt , Dt dan Σt didefisikan secara berturut-turut

$$\boldsymbol{\eta}_t = \left(\eta_1(t), \cdots, \eta_N(t)\right)$$

dengan

$$\eta_i(t) = \frac{\varepsilon_i(t)}{\sqrt{h_i(t)}} = \frac{\varepsilon_i(t)}{\sigma_i(t)}$$

atau

$$\varepsilon_i(t) = \sqrt{h_i(t)}\ \eta_i(t)$$

maka untuk setiap i (tetap), barisan variabel acak

{ηi (1), ... , ηi(T)}

merupakan barisan i.i.d. dengan rataan nol dan variansi satu, sehingga diperoleh

$$\mathbf{D}_t = diag\left(\sqrt{h_1(t)}, \cdots, \sqrt{h_N(t)}\right)$$

$$\boldsymbol{\Sigma}_t = \mathbf{D}_t\mathbf{R}\mathbf{D}_t$$ [10]

Pada penelitian ini, model GSTAR yang dikaji dibatasi hanya pada orde-1, maka untuk kesederhanaan penulisan, parameter $\Phi10$ dan $\Phi11$ dituliskan sebagai

$\Phi10$ = diag $(\phi_{01},\cdots,\phi_{0N})$ dan $\Phi11$ = diag $(\phi_{11},\cdots,\phi_{1N})$.

Model (1) secara simultan direpresentasikan dalam bentuk model linier

Y = Xβ + ε (2)

Model representasi linier (2) dapat diuraikan menjadi N persamaan regresi ganda dinyatakan atas masing-masing lokasi, terpisah satu dengan lainnya. Misalkan, untuk i = 1, ... , N,

$$\mathbf{y}_i = \begin{pmatrix} z_i(1) \\ \vdots \\ z_i(T) \end{pmatrix},\quad \mathbf{z}_i = \begin{pmatrix} z_i(0) \\ \vdots \\ z_i(T-1) \end{pmatrix},\quad \mathbf{v}_i = \begin{pmatrix} V_i(0) \\ \vdots \\ V_i(T-1) \end{pmatrix},\quad \boldsymbol{\varepsilon}_i = \begin{pmatrix} \varepsilon_i(1) \\ \vdots \\ \varepsilon_i(T) \end{pmatrix}.$$

Maka (6) dinyatakan sebagai

$$\begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_i \\ \vdots \\ \mathbf{y}_N \end{pmatrix} = \begin{pmatrix} \mathbf{z}_1 & 0 & \cdots & & 0 & \mathbf{v}_1 & 0 & \cdots & & 0 \\ 0 & \ddots & & & & 0 & \ddots & & & \vdots \\ \cdots & & \mathbf{z}_i & & & \vdots & & \mathbf{v}_i & & \\ & & & \ddots & 0 & & & & \ddots & 0 \\ 0 & \cdots & & 0 & \mathbf{z}_N & 0 & \cdots & & 0 & \mathbf{v}_N \end{pmatrix} \begin{pmatrix} \phi_{01} \\ \vdots \\ \phi_{0i} \\ \vdots \\ \phi_{0N} \\ \phi_{11} \\ \vdots \\ \phi_{1i} \\ \vdots \\ \phi_{1N} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_1 \\ \vdots \\ \boldsymbol{\varepsilon}_i \\ \vdots \\ \boldsymbol{\varepsilon}_N \end{pmatrix} \quad (3)$$

Persamaan (3) dituliskan untuk masing-masing lokasi ke-i, yaitu

$$Y_i(t) = X_i(t)\boldsymbol{\beta}_i + \varepsilon_i(t) \quad (4)$$

dengan

$$X_i(t) = \left(Z_i(t-1) \quad V_i(t-1) \right)$$

$$\boldsymbol{\beta}_i = \left(\phi_{0i} \quad \phi_{1i} \right)' \quad (5)$$

Secara simultan untuk t = 1, ... , T, maka (4) dinyatakan sebagai

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\varepsilon}_i \quad (6)$$

dengan

$$\mathbf{X}_i = \begin{pmatrix} Z_i(0) & V_i(0) \\ \vdots & \vdots \\ Z_i(T-1) & V_i(T-1) \end{pmatrix}$$

$$\boldsymbol{\varepsilon}_i = \begin{pmatrix} \varepsilon_i(1) \\ \vdots \\ \varepsilon_i(T) \end{pmatrix}$$

Bentuk (6) merupakan model regresi ganda pada lokasi ke-i dengan parameter regresi pada (5). Berhubung karena $\varepsilon i(t)$ merupakan galat dari model GSTAR-GARCH pada waktu t di lokasi ke-i dengan variansi bersyaratnya heteroskedastik,

$$Var(\varepsilon_i^2(t) | F_{t-1}) = h_i(t) = \alpha_{0,i} + \alpha_{1,i}\varepsilon_i^2(t-1) + \beta_{0,i} + \beta_{1,i} h_i(t-1)$$

maka model (6) merupakan model Regresi-GARCH [11]. Oleh karena itu, penaksiran parameter model GSTAR(1;1)-GARCH(1) dikerjakan dengan cara yang analog pada penaksiran parameter pada masing-masing lokasi secara keseluruhan untuk i = 1, 2, ... , N.

…

D. **STATUS LUARAN**: Tuliskan jenis, identitas dan status ketercapaian setiap luaran wajib dan luaran tambahan (jika ada) yang dijanjikan pada tahun pelaksanaan penelitian. Jenis luaran dapat berupa publikasi, perolehan kekayaan intelektual, hasil pengujian atau luaran lainnya yang telah dijanjikan pada proposal. Uraian status luaran harus didukung dengan bukti kemajuan ketercapaian luaran sesuai dengan luaran yang dijanjikan. Lengkapi isian jenis luaran yang dijanjikan serta mengunggah bukti dokumen ketercapaian luaran wajib dan luaran tambahan melalui Simlitabmas mengikuti format sebagaimana terlihat pada bagian isian luaran

Luaran pada penelitian ini adalah sebuah paper Paper yang sudah dipublikasikan pada jurnal : International Journal of Recent Technology and Engineering (IJRTE), Volume-8 Issue-2S7, July 2019, dengan judul: Multivariate GARCH Model and Its Application to Bivariate Model.

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Multivariate GARCH Model and Its Application to Bivariate Model

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Abstract: Multivariate GARCH model is a development of the univariate GARCH model. The multivariate GARCH model can be viewed as a conditional heteroskedasticity model in a multivariate time series. This paper discusses the parameterization of covariance matrices such as Vech model representation, BEEK model and Constant Correlation model. For parameter estimation the maximum likelihood method is used. Furthermore, multivariate GARCH model application is applied for bivariate model.

Index Terms: Multivariate, GARCH, Maximum likelihood.

I. INTRODUCTION

The conventional time series model assumes that the variance error is constant over time. The assumption of constant variance is an ideal assumption rarely encountered in real situations, especially related to the financial field [1]. Therefore a model developed with the assumption of nonconstant variance is known as heteroskedasticity model [2]. The heteroskedastic model is not only in univariate form but also in multivariate form [3]. The Generalized of space time autoregressive (GSTAR) model with ARCH error has also been developed [4]. In this paper we describe the parameterization of covariance matrices for the multivariate GARCH model.

II. UNIVARIATE ARCH AND GARCH MODEL

In conventional time series models such as the autoregressive moving average (ARMA) model it is assumed that the error variance (ε_t) is constant, ie $Var(\varepsilon_t) = \sigma^2$. Suppose the conditional variance of ε_T is not constant, then the variance of Y_t conditional on Y_{t-1} is not constant , $Var(\varepsilon_t) = \sigma_t^2$. One strategy is to model conditional variance as AR(q) process through the preceding error square, ie:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \Lambda + \alpha_{t-q}\varepsilon_{t-q}^2 + \eta_t \quad (1)$$

With η_t is a white-noise process. For this reason, (1) is called an autoregressive conditional heteroscedastic (ARCH) model [2]. Engle then proposes a scheme in which heteroscedasticy depends on previous Y_t values, namely

$$Y_t = \eta_t\sqrt{h_t} \text{ and } h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 \quad (2)$$

with η_t iid $N(0,1)$. The values of $\alpha_0 > 0$ and $\alpha_1 > 0$. Then (2) is called the ARCH(1)model. Note that the variance of Y_t conditional on Y_{t-1} is

$$Var(Y_t|Y_{t-1}) = Var(\eta_t\sqrt{h_t}|Y_{t-1}) = h_t Var(\eta_t) = h_t$$

If it is related to the application of the model, let Y_t be inflation then a process ARCH(1) states that high inflation in the past period will result in great variance at the present time.

Furthermore, if the ARCH(q) process is included lag of σ_t^2 then obtained model GARCH (p, q) [5], namely

$$h_t = \alpha_0 + \beta_1 h_1 + \Lambda + \beta_p h_{t-p} + \alpha_1 Y_{t-1}^2 + \Lambda + \alpha_q Y_{t-b}^2 \quad (3)$$

where p denotes lag on σ_t^2 and q states lag on Y_t^2. Specifically for $p = 1$ and $q = 1$ obtained the GARCH model (1, 1) ie

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_1 h_{t-1} \quad (4)$$

In this case
$E(Y_t|F_{t-1}) = 0$, and $Var(Y_t|F_{t-1}) = E(Y_{t-1}^2|F_{t-1}) = h_t$.

III. MULTIVARIATE ARCH/GARCH MODEL

The expansion of the ARCH/GARCH univariate model into the m-variate model requires the conditions that random variables ε_t have m-dimension, zero mean and conditional variance-covariance matrices of ε_t depend on the elements of the information set of historical data.

Let $\{\eta_t\}$ be a random variable vector i.i.d. sized (m x 1) with the following characteristics

$$E(\eta_t) = 0$$
$$E(\eta_t\eta_t') = I_m$$
$$\eta_t \sim G(0, I_m)$$

with G is a continuous density function. Suppose $\{\varepsilon_t\}$ is a randomly sized (mx1) random vector

$$\varepsilon_t = \eta_t\sqrt{H_t}$$

where:

$$E_{t-1}(\varepsilon_t) = 0$$
$$E_{t-1}(\varepsilon_t\varepsilon_t') = H_t$$

and H_t is the positive definite matrix of size (mxm) and measured to the set of information F_{t-1}, ie σ-field generated by the past information: $\{\varepsilon_{t-1}, \varepsilon_{t-2}, \ldots\}$. Parameterization of H_t as an ARCH (GARCH) multivariate,

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Luaran tambahan adalah dua makalah yang sudah dipresentasikan pada konferensi internasional The 3rd ICOR pada tanggal 20 September 2018 dan The 4th ICOR 19 September 2019 di Manado, yang mana papernya akan dipublikasikan pada Jurnal Internasional.

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E. **PERAN MITRA:** Tuliskan realisasi kerjasama dan kontribusi Mitra baik *in-kind* maupun *in-cash* (jika ada). Bukti pendukung realisasi kerjasama dan realisasi kontribusi mitra dilaporkan sesuai dengan kondisi yang sebenarnya. Bukti dokumen realisasi kerjasama dengan Mitra diunggah melalui Simlitabmas mengikuti format sebagaimana terlihat pada bagian isian mitra

Mitra dalam penelitian ini adalah tempat pengambilan data yaitu kantor BMKG Bitung. Peranan mitra adalah sebagai penyedia data yang diperlukan dalam penelitian ini yaitu data tinggi gelombang dan kecepatan angin di perairan laut Manado dan Bitung

F. **KENDALA PELAKSANAAN PENELITIAN**: Tuliskan kesulitan atau hambatan yang dihadapi selama melakukan penelitian dan mencapai luaran yang dijanjikan, termasuk penjelasan jika pelaksanaan penelitian dan luaran penelitian tidak sesuai dengan yang direncanakan atau dijanjikan.

Dalam analisis data, program komputer belum tersedia. Oleh karena itu disusun program komputer untuk analisis data menggunakan program S-Plus 8. Dalam penyusunan koding analisis data, sering mengalami program error, tetapi dengan mengulang dan meneliti setiap langkah, kesulitan dapat diatasi.

G. **RENCANA TINDAK LANJUT PENELITIAN:** Tuliskan dan uraikan rencana tindaklanjut penelitian selanjutnya dengan melihat hasil penelitian yang telah diperoleh. Jika ada target yang belum diselesaikan pada akhir tahun pelaksanaan penelitian, pada bagian ini dapat dituliskan rencana penyelesaian target yang belum tercapai tersebut.

Menyelesaikan penulisan draf paper yang ke dua untuk di submit ke jurnal internasional

H. DAFTAR PUSTAKA: Penyusunan Daftar Pustaka berdasarkan sistem nomor sesuai dengan urutan pengutipan. Hanya pustaka yang disitasi pada laporan akhir yang dicantumkan dalam Daftar Pustaka.

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Multivariate GARCH Model and Its Application to Bivariate Model

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Abstract*: *Multivariate GARCH model is a development of the univariate GARCH model. The multivariate GARCH model can be viewed as a conditional heteroskedasticity model in a multivariate time series. This paper discusses the parameterization of covariance matrices such as Vech model representation, BEEK model and Constant Correlation model. For parameter estimation the maximum likelihood method is used. Furthermore, multivariate GARCH model application is applied for bivariate model.

Index Terms*: *Multivariate, GARCH, Maximum likelihood.

I. INTRODUCTION

The conventional time series model assumes that the variance error is constant over time. The assumption of constant variance is an ideal assumption rarely encountered in real situations, especially related to the financial field [1]. Therefore a model developed with the assumption of nonconstant variance is known as heteroskedasticity model [2]. The heteroskedastic model is not only in univariate form but also in multivariate form [3]. The Generalized of space time autoregressive (GSTAR) model with ARCH error has also been developed [4]. In this paper we describe the parameterization of covariance matrices for the multivariate GARCH model.

II. UNIVARIATE ARCH AND GARCH MODEL

In conventional time series models such as the autoregressive moving average (ARMA) model it is assumed that the error variance (ε_t) is constant, ie $Var(\varepsilon_t) = \sigma^2$. Suppose the conditional variance of ε_t is not constant, then the variance of Y_t conditional on Y_{t-1} is not constant $,Var(\varepsilon_t) = \sigma_t^2$. One strategy is to model conditional variance as AR(q) process through the preceding error square, ie:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \Lambda + \alpha_{t-q}\varepsilon_{t-q}^2 + \eta_t \qquad (1)$$

With η_t is a white-noise process. For this reason, (1) is called an autoregressive conditional heteroscedastic (ARCH) model [2]. Engle then proposes a scheme in which heteroscedasticy depends on previous Y_t values, namely

$$Y_t = \eta_t\sqrt{h_t} \text{ and } h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 \qquad (2)$$

with η_t iid. $N(0.1)$. The values of $\alpha_0 > 0$ and $\alpha_1 > 0$. Then (2) is called the ARCH(1)model. Note that the variance of Y_t conditional on Y_{t-1} is

$$Var(Y_t|Y_{t-1}) = Var(\eta_t\sqrt{h_t}|Y_{t-1}) = h_t Var(\eta_t) = h_t$$

If it is related to the application of the model, let Y_t be inflation then a process ARCH(1) states that high inflation in the past period will result in great variance at the present time.

Furthermore, if the ARCH(q) process is included lag of σ_t^2 then obtained model GARCH (p, q) [5], namely

$$h_t = \alpha_0 + \beta_1 h_1 + \Lambda + \beta_p h_{t-p} + \alpha_1 Y_{t-1}^2 + \Lambda + \alpha_q Y_{t-9}^2 \qquad (3)$$

where p denotes lag on σ_t^2 and q states lag on Y_t^2. Specifically for p = 1 and q = 1 obtained the GARCH model (1, 1) ie

$$h_t = \alpha_0 + \alpha_1 Y_{t-1}^2 + \beta_1 h_{t-1} \qquad (4)$$

In this case

$E(Y_t|F_{t-1}) = 0$, and $Var(Y_t|F_{t-1}) = E(Y_{t-1}^2|F_{t-1}) = h_t$.

III. MULTIVARIATE ARCH/GARCH MODEL

The expansion of the ARCH/GARCH univariate model into the m-variate model requires the conditions that random variables $\boldsymbol{\varepsilon}_t$ have m-dimension, zero mean and conditional variance-covariance matrices of $\boldsymbol{\varepsilon}_t$ depend on the elements of the information set of historical data.

Let $\{\boldsymbol{\eta}_t\}$ be a random variable vector i.i.d. sized (m x 1) with the following characteristics

$$E(\boldsymbol{\eta}_t) = 0$$
$$E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') = \mathbf{I}_m$$
$$\boldsymbol{\eta}_t \sim G(0,\mathbf{I}_m)$$

with G is a continuous density function. Suppose $\{\boldsymbol{\varepsilon}_t\}$ is a randomly sized (mx1) random vector

$$\varepsilon_t = \eta_t\sqrt{H_t}\ .$$

where:

$$E_{t-1}(\varepsilon_t) = 0$$
$$E_{t-1}(\varepsilon_t\varepsilon_t') = H_t$$

and H_t is the positive definite matrix of size (mxm) and measured to the set of information F_{t-1}, ie σ-field generated by the past information: $\{\varepsilon_{t-1}, \varepsilon_{t-2}, ...\}$. Parameterization of H_t as an ARCH (GARCH) multivariate,

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as a function of the information set of F_{t-1}, then the elements of H_t are dependent on the lag-q of the ε_t and cross-products ε_t squares.

Thus the elements of the covariance matrix follow an ARMA process vector in the square and cross-products of disturbances (error). Parameterization of H_t as a multivariate ARCH (GARCH) is given in three forms, ie vech model, BEKK model and consonant correlation model [3].

A. Vech Representation

Vech is the vector-half operator (ie half-vector) which is piling the elements of the lower triangle of the $m\times m$ matrix into the vector sized $(m(m+1)/2)\times 1$. Vech representation is often called full parameterization. Since the covarians matrix H_t is a symmetric matrix, then vech (H_t) contains elements in H_t singly. Thus, the expansion of GARCH multivariate model (p,q) in vech representation [3] can be written as

$$vech(H_t) = W + \sum_{i=1}^{q} A_i vech(\varepsilon_{t-1}\varepsilon_{t-1}) + \sum_{j=1}^{p} B_j vech(H_{t-j}) \qquad (5)$$

where: W is a vector of sizes $(m(m+1)/2)\times 1$, whereas A_i and B_j are matrices $(m(m+1)/2)\times(m(m+1)/2)$. The number of parameters in the general formulation of vech representation is as much as $\{m(m+1)/2+(p+q)(m(m+1)/2)^2\}$. For example, let $m = 2$, and $p = q = 1$, then the number of parameters is 21 pieces. The form of the *vech* (H_t) model for this example is as follows:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}$$

In this case, the element to (i, j) in H_t depends on the (i, j) element corresponding in $\varepsilon_t\varepsilon_t'$ and H_{t-1}. To ensure that the H_t matrix is positive definite, the necessary conditions [3] are

$$\begin{aligned} & w_1 > 0,\ w_3 > 0,\ w_1 w_3 - w_2^2 > 0, \\ & a_{11} \geq 0,\ a_{13} \geq 0,\ a_{31} \geq 0,\ a_{33} \geq 0,\ a_{11}a_{33} - a_{22}a_{22} \geq 0, \\ & a_{11}a_{13} - (1/4)\, a_{12}a_{12} \geq 0,\ a_{11}a_{31} - a_{21}a_{21} \geq 0, \\ & a_{31}a_{33} - (1/4)\, a_{32}a_{32} \geq 0,\ a_{13}a_{33} - a_{23}a_{23} \geq 0. \end{aligned} \qquad (6)$$

To reduce the large number of parameters then simplified the vech representation. One way is to select the matrices A and B in diagonal form [3]. This is called the *vech diagonal* model. This simplification reduces the number of parameters where many parameters become $(m(m+1)/2)(1+p+q)$. Suppose that for m=2, and p=q=1, then the diagonal vech model [3] can be written as:

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \qquad (7)$$

By multiplying the above matrix is obtained

$$\begin{aligned} h_{11,t} &= w_1 + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1} \\ h_{21,t} &= w_2 + a_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{22}h_{21,t-1} \\ h_{22,t} &= w_3 + a_{33}\varepsilon_{2,t-1}^2 + b_{33}h_{22,t-1} \end{aligned} \qquad (8)$$

In this case the number of parameters is reduced from 21 simplified to only 9. By subtracting this parameter, the conditions for the positive definite matrix H_t [3] are

$$\begin{aligned} & w_1 > 0,\ w_3 > 0,\ w_1 w_3 - w_2^2 > 0, \\ & a_{11} \geq 0,\ a_{33} \geq 0,\ a_{11}a_{33} - a_{22}a_{22} \geq 0. \end{aligned} \qquad (9)$$

B. Positive Definite Parameterization

This positive definite parameterization is also known as BEKK (Baba, Engle, Kraft and Kroner). The parameterization model from BEKK [3] is

$$h_t = CC' + \sum_{k=1}^{k}\sum_{i=1}^{q} A_{ik}\varepsilon_{t-1}\varepsilon_{t-1}' A_{ik}' + \sum_{k=1}^{k}\sum_{i=1}^{p} B_{ik} H_{t-i} B_{ik}' \qquad (10)$$

where C is the lower triangular matrix, so CC' is the symmetric parameter matrix (mxm), whereas A_{ik} and B_{ik} are any parameter matrices (mxm). This model assures that the H_t matrix is a positive definite matrix. For example, take K=p=q=1 and N=2. Then the above BEKK model is

$$\begin{aligned} \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} &= \begin{pmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{pmatrix} \\ &+ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \\ &+ \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \end{aligned} \qquad (11)$$

Then by multiplying the matrix is obtained

$$h_{11,t} = c_{11}^2 + (a_{11}\varepsilon_{1,t-1} + a_{12}\varepsilon_{2,t-1})^2 + (b_{11}h_{11,t-1} + b_{12}h_{22,t-1})^2$$

$$h_{22,t} = c_{21}^2 c_{22}^2 + (a_{21}\varepsilon_{1,t-1} + a_{22}\varepsilon_{2,t-1})^2 + (b_{21}h_{12,t-1} + b_{22}h_{22,t-1})^2$$

$$\begin{aligned} h_{21,t} = h_{12,t} &= c_{11}c_{21} + a_{11}a_{21}\varepsilon_{1,t-1} \\ &+ (a_{11}a_{22} + a_{12}a_{21})\varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ &+ a_{12}a_{22}\varepsilon_{2,t-1}^2 + b_{11}b_{21}h_{11,t-1} \\ &+ (b_{11}b_{22} + b_{12}b_{21})h_{21,t-1} \\ &+ b_{21}b_{22}h_{22,t-1} \end{aligned} \qquad (12)$$

C. Constant Correlation Model

In a constant correlation model, conditional covariance is proportional to the corresponding conditional standard deviation. In this case, it is assumed that the correlation matrix $\boldsymbol{\varepsilon}_t$, conditional to the past, is constant. This assumption simplifies the calculation load in the estimate and also the requirement that a positive definite H_t for each t be easily obtained. So the assumptions in this model [3] are

$$E_{t-1}[\varepsilon_t] = 0$$

$$E_{t-1}[\varepsilon_t\varepsilon_t'] = H_t$$

$$\{H_t\}_{ii} = h_{ii,t}$$

$$\{H_t\}_{ij} = h_{ij,t} = \rho_{ij}\sqrt{h_{ii,t}h_{jj,t}} \qquad if \quad i \neq j \qquad (13)$$

Let $\mathbf{D}_t$ denote the mxm diagonal matrix with its diagonal element is the conditional variance, ie $\{D_t\}_{ii} = h_{ii,t}$. Let $\mathbf{R}_t$ denote a constant correlation matrix in which the *ij*-element is

$$\{R_t\}_{ij} = \{H_t\}_{ij}(\{H_t\}_{ii}\{H_t\}_{jj})^{-1/2} \quad i,j = 1,2,\ldots,m \qquad (14)$$

Assuming the model $\mathbf{R}_t = \mathbf{R}$, then

$$\mathbf{H}_t = \mathbf{D}_t^{1/2}\mathbf{R}_t\mathbf{D}_t^{1/2}$$

$$H_t = diag(\sqrt{h_{11,t}},\ldots,\sqrt{h_{mm,t}})R_t diag(\sqrt{h_{11,t}},\ldots,\sqrt{h_{mm,t}})$$

For example, for $N = 2$, $p = q = 1$.Then

$$H_t = \begin{pmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{pmatrix}\begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix}\begin{pmatrix} \sqrt{h_{11,t}} & 0 \\ 0 & \sqrt{h_{22,t}} \end{pmatrix} \qquad (15)$$

In this case,

$$D_t = \begin{pmatrix} h_{11,t} & 0 \\ 0 & h_{22,t} \end{pmatrix}, \text{ and } R_t = \begin{pmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{pmatrix},$$

so

$$H_t = \begin{pmatrix} h_{11,t} & \rho\sqrt{h_{11,t}h_{22,t}} \\ \rho\sqrt{h_{11,t}h_{22,t}} & h_{22,t} \end{pmatrix}$$

where $\rho = \rho_{12} = \rho_{21}$ is the correlation coefficient between $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ and satisfies the correlation properties $|\rho| < 1$, while $h_{11,t}$ and $h_{22,t}$ are the variance for standard GARCH(p,q) processes. Suppose for $p = q = 1$, then

$$h_{11,t} = \alpha_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}$$

$$h_{22,t} = \alpha_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}$$

$$h_{12,t} = h_{21,t}$$

$$= \rho\sqrt{\alpha_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}}\sqrt{\alpha_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}}$$

The number of parameters to be estimated is 6. To qualify where the H_t matrix is positive definite then in this constant correlation model we need the following conditions [3]:

$$\alpha_{i0} > 0, \quad a_{ij} \geq 0 \text{ dan } b_{ik} \geq 0,$$
$$i = 1, \ldots, m, \quad j = 1, \ldots, q, \quad \text{and } k = 1, \ldots, p.$$

If the conditional variance along the diagonal of the matrix **D** is all positive, the correlation matrix R is a positive definite and this results in the sequence of conditional covariance matrices $\{H_t\}$ making sure the positively definite is almost certain for all t. Next inverse of H_t is:

$$\mathbf{H}_t^{-1} = \mathbf{D}_t^{-1/2}\mathbf{R}^{-1}\mathbf{D}_t^{-1/2}.$$

IV. PARAMETER ESTIMATION

Suppose $\{\mathbf{y}_t\}$, $t = 1, 2, \ldots, T$, is the realization of the stochastic process vector, with the mean vector $\boldsymbol{\mu}_t = \mathbf{0}$ and the conditional covariance matrix is $\mathbf{H}_t(\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the parameter vector. Multivariate normal density of the process vector [1,6] is

$$f(y_t|F_{k-1}) = \frac{1}{\sqrt{(2\pi)^N}|H_t|^{1/2}}\exp\left\{\frac{-1}{2}y_t'H_t^{-1}y_t\right\}, \qquad (16)$$
$$t = 1,2,\ldots,T.$$

The multivariate ARCH(GARCH) model parameter estimation uses the likelihod maximum method. The log-likelihood function for normal multivariate combined density [7] is

$$L(\theta) = -\frac{TN}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}|H_t| - \frac{1}{2}\sum_{t=1}^{T}y_t'H_t^{-1}y_t \qquad (17)$$

Next we can estimate **θ** by completing the derivative of the log likelihood function $\partial L/\partial\theta = 0$. The parameter estimation for the diagonal vech model is as follows:

$$H_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix}$$

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}\begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}\begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix} \qquad (18)$$

Parameters $\boldsymbol{\theta} = (w_1, w_2, w_3, a_{11}, a_{22}, a_{33}, b_{11}, b_{22}, b_{33})'$, ie there are 9 parameters to be estimated. Solution $\partial L/\partial\theta = 0$ is solving simultaneously 9 equations, namely:

$$\frac{\partial L}{\partial w_1} = 0, \frac{\partial L}{\partial w_2} = 0, \frac{\partial L}{\partial w_3} = 0, \frac{\partial L}{\partial a_{11}} = 0, \frac{\partial L}{\partial a_{22}} = 0, \frac{\partial L}{\partial a_{33}} = 0, \qquad (19)$$
$$\frac{\partial L}{\partial b_{11}} = 0, \frac{\partial L}{\partial b_{22}} = 0, \frac{\partial L}{\partial b_{33}} = 0.$$

Compared with the constant correlation model, the parameter estimation is as follows

$$H_t = \begin{pmatrix} h_{11,t} & \rho\sqrt{h_{11,t}h_{22,t}} \\ \rho\sqrt{h_{11,t}h_{22,t}} & h_{22,t} \end{pmatrix}$$

$$h_{11,t} = a_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}$$

$$h_{22,t} = a_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}$$

$$h_{12,t} = h_{21,t}$$

$$= \rho\sqrt{a_{10} + a_{11}\varepsilon_{1,t-1}^2 + b_{11}h_{11,t-1}}\sqrt{a_{20} + a_{21}\varepsilon_{2,t-1}^2 + b_{21}h_{22,t-1}}$$

Parameter $\boldsymbol{\theta} = (a_{10}, a_{11}, b_{11}, a_{20}, a_{21}, b_{21})'$. Solution $\partial L / \partial\theta = 0$ is solving simultaneously 6 equations, namely

$$\frac{\partial L}{\partial a_{10}} = 0, \frac{\partial L}{\partial a_{11}} = 0, \frac{\partial L}{\partial a_{20}} = 0, \qquad (20)$$
$$\frac{\partial L}{\partial a_{21}} = 0, \frac{\partial L}{\partial b_{11}} = 0, \frac{\partial L}{\partial b_{21}} = 0$$

V. APPLICATION TO BIVARIATE MODEL

The application of the multivariate GARCH model in this paper is given for the bivariate model.

The data taken are weekly average price data of 2017 for chili and onion in Manado city. The amount of data is 52 data. The result of ARCH effect test showed that both data had significant ARCH effect with *p*-value of 0.0023 and 0.0005 respectively.

The result of the parameter estimation for the vech representation bivariate model with the help of computer softwear [8] is

```
> ca.ba.dvec = mgarch(ca.ba~1, ~dvec(1,1), trace=F)
Coefficients:
        C(1) 6.154e+004
        C(2) 3.664e+004
     A(1, 1) 3.742e+007
     A(2, 1) 8.265e+006
     A(2, 2) 4.086e+006
 ARCH(1; 1, 1) 1.000e-001
 ARCH(1; 2, 1) 1.000e-001
 ARCH(1; 2, 2) 1.000e-001
GARCH(1; 1, 1) 8.100e-001
GARCH(1; 2, 1) 8.100e-001
GARCH(1; 2, 2) 8.100e-001
```

Then the vech representation bivariate model (7) is obtained, namely

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} 3.742e+007 \\ 8.265e+006 \\ 4.086e+006 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} 0.81 & 0 & 0 \\ 0 & 0.81 & 0 \\ 0 & 0 & 0.81 \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{21,t-1} \\ h_{22,t-1} \end{bmatrix}$$

$$\begin{bmatrix} h_{11,t} \\ h_{21,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} 3.742e+007 + 0.1\varepsilon^2_{1,t-1} + 0.81h_{11,t-1} \\ 8.265e+006 + 0.1\varepsilon_{1,t-1}\varepsilon_{2,t-1} + 0.81h_{21,t-1} \\ 4.086e+006 + 0.1\varepsilon^2_{2,t-1} + 0.81h_{22,t-1} \end{bmatrix} \quad (21)$$

VI. CONCLUSION

Representation of multivariate ARCH (GARCH) can be given in three forms, ie Vech, BEKK and Consonant Correlation model. The *Vech diagonal* model reduces the number of parameters thus simplifying the vech representation. The multivariate GARCH model can be applied to the data of chili and onion because the test of effect arch on the data is significant.

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GSTAR-GARCH Model and The Application

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Abstract.

Generalisasi Space Time Autoregressive (GSTAR) models was introduced in Ruchjana (2002) assumed constant variance errors (homoscedastic). In this paper, we consider GSTAR models with GARCH errors (GSTAR-GARCH. The variance of error is not constant, it changes over time (heteroscedastic), hence the term is heteroscedastic GSTAR model.The conditional variance of errors conditions on the past changed over time but the unconditional variance was constant. The error terms as a multivariat GARCH is modeled by constant conditional correlations models. The least squared method used to estimated the mean equation (GSTAR) parameters then the error term (GARCH) parameters estimated by maximum likelihood method. We apply the GSTAR-GARCH model using simulation data in three locations.

Keywords: Gstar model, arch, garch, heteroscedastic.

INTRODUCTION

GSTAR models are often useful in modeling time series involved time variable and space-time variable. However, these model have the assumption of homoscedastic (or equal variance) for the errors. This is not appropriate when dealing with the financial market variables such as the stock price indices or currency exchange rates. In the real situation we often find that the variance changes over all time *t*. The financial market variables typically have characteristics which the assumption of homoscedastic have failed to consider. These typical are the unconditional distribution of financial time series has heavier tails than the normal distribution, Values of ε_t do not have much correlation, but values of ε_t square are highly correlated and the changes in ε_t tend to cluster, large (or small) changes in ε_t tend to be followed by large (or small) changes. So that, in the financial market variables the assumption of heteroscedastic more appropriate. One of the time series models allowing for heteroscedasticity is the Autoregressive Conditional Heteroscedastic (ARCH) model introduced by Engle [5]. In this paper, we will study about the orde-1 GSTAR-GARCH model. The estimation of parameters concist of two parts, the least squared method used to estimate the mean equation (GSTAR) parameters then the error term (GARCH) parameters are estimated using maximum likelihood method.

GSTAR models is a development of the model of the Space Time Autoregressive (STAR), because the GSTAR can caught a phenomenon with locations that has heterogeneous characteristics and the parameters for each location different from each other. But, the STAR model assumes that the locations have homogeneous characteristics. GSTAR model has focus to predict the mean conditional whereas variance is assumed to be constant over time. The assumption of constant variance is ideal assumption that rarely found in practice. In a real situation often found a phenomenon that has a variance always changing. For example, the phenomenon of inflation in general has a variance is not constant. Therefore, modeling GSTAR when applied to inflation, we need to develop an assumption of error from constant variance into non constant variance. GSTAR by assuming constant variance can not to catch a real phenomenon involving nonconstant variance. The GSTAR model with the error has non constant variance is expressed with a model named GSTAR ARCH-ARCH developed by Nainggolan (2010; 2011) . Thus, GSTAR-ARCH can be obtained the prediction of the conditional mean and conditional variance. The parameters estimation can be solve by maximum likelihood and then generalized least squares (Greene, 2002). The objectives of this paper are (1) To introduce the GSTAR-GARCH Models, (2) To study the parameter estimation of the order-1 GSTAR-GARCH model and (3) To give an illustration about the GSTAR-GARCH model.

GSTAR MODELS

Let $\mathbf{z}(1), \mathbf{z}(2), \cdots\cdots, \mathbf{z}(T)$ are the time series observations. The process $\{\mathbf{z}(t)\}$ with

$$\mathbf{z}(t) = (Z_1(t), \cdots\cdots, Z_N(t))'$$

is an order-1 Generalized Space Time Autoregressive models, GSTAR(1,1), that is order-1 at lag-time and order-1 at lag-spacial, if statisfy

$$\mathbf{z}(t) = \mathbf{\Phi}_{10}\mathbf{z}(t-1) + \mathbf{\Phi}_{11}\mathbf{W}\mathbf{z}(t-1) + \boldsymbol{\varepsilon}(t)$$
$$\mathbf{z}(t) = \left[\Phi_{10}\mathbf{I} + \Phi_{11}\ \mathbf{W}\ \right]\mathbf{z}(t-1) + \boldsymbol{\varepsilon}(t)$$
$$\mathbf{z}(t) = \Phi\mathbf{z}(t-1) + \boldsymbol{\varepsilon}(t) \quad (1)$$

where

$$\boldsymbol{\varepsilon}(t) \overset{iid}{\sim} N(0, \sigma^2\mathbf{I}_N)$$
$$\Phi = \left[\Phi_{10}\mathbf{I} + \Phi_{11}\ \mathbf{W}\ \right]$$

W is weight matrix.

The assumption in (8) is that the error variance is constant every time (homoscedasticity). To estimates the parameters of these models, make the models in linear form

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \quad (2)$$

where $\boldsymbol{\varepsilon} \overset{iid}{\sim} N(0, \sigma^2\mathbf{I}_{TN})$. The least square estimation of the parameters used the formula [8]:

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} \quad (3)$$

GSTAR models is the development of a model of the Space Time Autoregressive (STAR). GSTAR can caught a phenomenon with locations that have heterogeneous characteristics, because of the parameters for each location different from each other. So, it is a generalized of the STAR model that assumes for locations with homogeneous characteristics.

Suppose $Z_1, Z_2, \ldots, Z_T$ are the time series observation. The process $\{ Z_t \}$ is an GSTAR $(p_{\lambda_1,\lambda_2,\cdots,\lambda_p})$ models of autoregressive order p and spatial order λ_s of the sth autoregressive term is form (Borovkova, 2008)

$$\mathbf{Z}(t) = \sum_{s=1}^{p}[\Phi_{s0}\mathbf{Z}(t-s) + \sum_{k=1}^{\lambda_s}\Phi_{sk}\mathbf{W}^{(k)}\mathbf{Z}(t-s)] + \boldsymbol{\varepsilon}(t) \quad (4)$$

where p is autoregressive order, λ_s is space-time order of *s-term* of autoregressive,

$$\Phi_{sk} = diag(\phi_{sk}^{(1)}, \cdots, \phi_{sk}^{(N)})$$

and $\mathbf{W}^{(k)}$ is a weight matrix. The model (1) assumes that

$$\boldsymbol{\varepsilon}_t \sim iid.\, N(0, \sigma^2\mathbf{I}_N).$$

By this assumption, we can say that the vector of error $\boldsymbol{\varepsilon}(t)$ in (1) is uncorrelated with the previously $\boldsymbol{\varepsilon}(t-i)$, $i = 1, 2, \ldots$. Also, for any $i \neq j$ the location $\varepsilon_i(t)$ and $\varepsilon_j(t)$ are not correlated. Then, the error has normal ditribution, identic and independent.

GSTAR (1_1) model, for example is:

$$\mathbf{Z}(t) = \mathbf{\Phi}_{10}\mathbf{Z}(t-1) + \mathbf{\Phi}_{11}\mathbf{W}\ \mathbf{Z}(t-1) + \boldsymbol{\varepsilon}(t)$$
$$\mathbf{Z}(t) = [\mathbf{\Phi}_{10} + \mathbf{\Phi}_{11}\mathbf{W}]\mathbf{Z}(t-1) + \boldsymbol{\varepsilon}(t)$$
$$\mathbf{Z}(t) = \mathbf{\Phi}\mathbf{Z}(t-1) + \boldsymbol{\varepsilon}(t)$$

where $\mathbf{\Phi} = [\Phi_{10}\mathbf{I} + \Phi_{11}\mathbf{W}]$. The GSTAR model parameters can be estimated by first, make the GSTAR models in linear form

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

and then least square method to estimate the parameters by $\boldsymbol{\beta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ (Ruchjana, 2002)).

ARCH AND GARCH MODELS

Suppose that $\varepsilon_1, \varepsilon_2, \cdots\cdots, \varepsilon_T$ are time series observations and let F_t be the set of ε_t up to time *t*, including ε_t for $t \le 0$. The process $\{\varepsilon_t\}$ is an Autoregressive Conditional Heteroscedastic process of order-*q*, ARCH(*q*) models [5], if statisfy

$$\varepsilon_t | F_{t-1} \sim N(0, h_t) \quad (5)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots\cdots + \alpha_q \varepsilon_{t-q}^2 \quad (6)$$

wiht $q > 0$, $\alpha_0 > 0$, and $\alpha_i \ge 0$, for $i = 1, 2, \ldots\ldots, q$.
From (1),

$$E(\varepsilon_t | F_{t-1}) = 0$$

$$Var(\varepsilon_t | F_{t-1}) = E(\varepsilon_t^2 | F_{t-1}) = h_t \cdot$$

The process $\{\varepsilon_t\}$ is a Generalized Autoregressive Conditional Heteroscedastic process of order *p* and *q*, GARCH(*p*,*q*) models [1], if :

$$\varepsilon_t | F_{t-1} \sim N(0, h_t)$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots\cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_j h_{t-1} + \cdots\cdots + \beta_p h_{t-p} \quad (7)$$

where $q > 0$, $\alpha_0 > 0$, and $\alpha_i \ge 0$, for $i = 1, 2, \ldots\ldots, q$, $\beta_j \ge 0$, for $j = 1, 2, \ldots\ldots, p$.

Next, the process $\{\varepsilon_t\}$ from (1) devided by the square root of the conditional variance in (2) or (3), we have:

$$\left(\varepsilon_t / \sqrt{h_t}\right) \Big| F_{t-1} \sim N(0,1)$$

and therefore the sequence η_1 , ... , η_T , defined by

$$\eta_t = \varepsilon_t / \sqrt{h_t}$$

$$\varepsilon_t = \eta_t \sqrt{h_t} \quad (8)$$

should be independent and identically distributed *i.i.d.* $N(0,1)$. So, the ARCH process $\{\varepsilon_t\}$ can be constructed from a sequence of i.i.d $N(0,1)$ random variables in (4).

In general, the time series models can be written in the form

$$Z_t = f(X_t, t-1) + \varepsilon_t \quad (9)$$

where Z_t is time series at the time *t*, $f(X_t, t\text{-}1)$ is mean equations, and ε_t is variance (innovation) equations of the error. For example, let the mean equations in (5) is ordinary regression models and the variance equations is ARCH models, so it is called the ARCH regression models, see [5]. The ARCH(*q*) regression models can be written as

$$Z_t | F_{t-1} \sim N(\mathbf{X_t}\boldsymbol{\beta}, h_t)$$

$$Z_t = \mathbf{X}_t\boldsymbol{\beta} + \varepsilon_t \quad (10)$$

$$\varepsilon_t = \eta_t \sqrt{h_t}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots\cdots + \alpha_q \varepsilon_{t-q}^2 \quad (11)$$

where $\alpha_0 > 0$, $\alpha_i \ge 0$, $i = 1, \cdots, q$, $\eta_t \sim iid.\ N(0,1)$.

The specific of regression-ARCH models is that the errors staisfy the ARCH process. In this models, $\mathbf{X}_t$ may include lagged dependent and exogenous variables. The ordinary least squares (OLS) estimator of $\boldsymbol{\beta}$ is still consistent as $\mathbf{X}$ and ε are uncorrelated throught the difinition of the regression as a conditional expectation [5]. The procedure to estimate the parameters is to initially estimate $\boldsymbol{\beta}$ by OLS and then obtain the residuals. From these residuals, estimate the parameters in (7), $\hat{\alpha} = (\alpha_0, \alpha_1, \cdots, \alpha_p)'$, and based on these $\hat{\alpha}$, efficient estimated of $\boldsymbol{\beta}$ are found, see [5].

Then, the process { ε_t } is an Generalized Autoregressive Conditional Heteroscedastic process of order p and q, GARCH(p,q), if statisfy [Bollerslev, 1986]

$$h_t = \alpha_0 + \alpha_1\, \varepsilon_{t-1}^2 + \cdots + \alpha_q\, \varepsilon_{t-q}^2 + \beta_1\, h_{t-1} + \cdots + \beta_p\, h_{t-p}$$

where $q > 0$, $\alpha_0 > 0$, and $\alpha_i \geq 0$, for $i = 1, 2, \ldots\ldots, q$, $\beta_j \geq 0$, for $j = 1, 2, \ldots\ldots, p$.

Next, dividing by the square root of the conditional variance of ε_t , we have:

$$\frac{\varepsilon_t}{\sqrt{h_t}} \Big| F_{t-1} \sim N(0,1)$$

and therefore the sequence η_1 , …… , η_T, defined by $\eta_t = \dfrac{\varepsilon_t}{\sqrt{h_t}}$ or $\varepsilon_t = \eta_t \sqrt{h_t}$ is a iid. sequence with zerro mean and variance 1.

MAIN RESULTS

Time series models, in general can be discribed in two parts, namely the mean and variance (innovation) equations, see eq. (2). Therefore, the development of GSTAR can be seen from these: the mean and variance .

$$\mathbf{Z}_t = f(X_t, t-1) + \boldsymbol{\varepsilon}(t) \tag{12}$$

In development of the mean equations, GSTAR model can be developed into a model GSTARI, GSTARIMA, GSTAR SUR and others. In this models, the assumtions are contant variance (homoscedastic). On the other hand, GSTAR model can developed from the variance equation, that is the variance not contant over time (heteroscedastic). If a phenomenom statisfies heteroscedastic conditions we can make GSTAR model with non constant variance (Fig.1). The variance can be make in the simplest form as a ARCH model (Engle, 1982). Then, Bollerslev and the others develop the ARCH model into GARCH, EGARCH, NARCH, TARCH and IGARCH models.

FIGURE 1. The Diagram of Development of a GSTAR Models with Heteroscedastic Variance

In this paper, we give GSTAR-ARCH models restricted at the orde-1 that is GSTAR(1_1) models and multivariat ARCH(1) models. Let $\mathbf{Z}(t) = (Z_1(t), \ldots , Z_N(t))'$ is a time series vector, $E[\mathbf{Z}(t)] = \mathbf{0}$, dan $\boldsymbol{\varepsilon}(t) = (\varepsilon_1(t), \ldots , \varepsilon_N(t))'$,is i.i.d. random vectors, zero mean dan constant variance. Then GSTAR(11)-ARCH(1) Model is

$$\begin{aligned} &\mathbf{Z}(t) = \mathbf{\Phi}_{10}\mathbf{Z}(t-1) + \mathbf{\Phi}_{11}\mathbf{W}\,\mathbf{Z}(t-1) + \boldsymbol{\varepsilon}(t) \\ &\boldsymbol{\varepsilon}(t) = \mathbf{D}_t \boldsymbol{\eta}_t \\ &(\boldsymbol{\varepsilon}(t) | \mathbf{F}_{t-1}) \sim N(0, \mathbf{\Sigma}(t)) \end{aligned} \tag{13}$$

where

$$\mathbf{D}_t = diag(\sqrt{h_1(t)}, \cdots, \sqrt{h_N(t)}\ )$$

$$\boldsymbol{\eta}_t = (\eta_1(t), \cdots, \eta_N(t))' = \left(\frac{\varepsilon_1(t)}{\sqrt{h_1(t)}}, \cdots, \frac{\varepsilon_N(t)}{\sqrt{h_N(t)}} \right)$$

$$\Sigma(t) = \mathbf{D}_t \mathbf{R} \mathbf{D}_t \ .$$

The matrix **R** is constant conditional correlation matrix of errors $\varepsilon_i(t)$ and $\varepsilon_j(t)$. To get matrix R, we use the correlation

$$\rho_{ij} = \frac{E[\varepsilon_i(t)\varepsilon_j(t) | F_{t-1}]}{\sqrt{Var(\varepsilon_i(t))}\sqrt{Var(\varepsilon_j(t))}} = \frac{\sigma_{ij}(t)}{\sqrt{h_i(t)}\sqrt{h_j(t)}}$$

or

$$\sigma_{ij}(t) = \rho_{ij}\sqrt{h_i(t)}\sqrt{h_j(t)}$$

In this paper, we assume the correlation in constant. The covariance matrix $\mathbf{\Sigma}(t)$ is

$$\Sigma(t) = \begin{pmatrix} h_1(t) & \rho_{12}\sqrt{h_1(t)}\sqrt{h_2(t)} & \cdots & \rho_{1N}\sqrt{h_1(t)}\sqrt{h_N(t)} \\ \rho_{21}\sqrt{h_2(t)}\sqrt{h_1(t)} & h_2(t) & \cdots & \rho_{2N}\sqrt{h_2(t)}\sqrt{h_N(t)} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1}\sqrt{h_N(t)}\sqrt{h_1(t)} & \rho_{N2}\sqrt{h_N(t)}\sqrt{h_2(t)} & \cdots & h_N(t) \end{pmatrix}$$

$$\Sigma(t) = \begin{pmatrix} \sqrt{h_1(t)} & 0 & \cdots & 0 \\ 0 & \sqrt{h_2(t)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{h_N(t)} \end{pmatrix} \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \rho_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & 1 \end{pmatrix} \begin{pmatrix} \sqrt{h_1(t)} & 0 & \cdots & 0 \\ 0 & \sqrt{h_2(t)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{h_N(t)} \end{pmatrix} = \mathbf{D}_t \mathbf{R} \mathbf{D}_t$$

To construct the GSTAR-ARCH model, we define the variance $h_i(t)$ as a ARCH(q) model, that is

$$\sigma_i^2(t) = h_i(t) = \alpha_{0,i} + \alpha_{1,i}\ \varepsilon_i^2(t-1) + \cdots + \alpha_{q,i}\ \varepsilon_i^2(t-q)\ .$$

In the similar way, to construct the GSTAR-GARCH model, we define the variance $h_i(t)$ as a GARCH(q) model, that is

$$h_i(t) = \alpha_{0,i} + \alpha_{1,i}\ \varepsilon_i^2(t-1) + \cdots + \alpha_{q,i}\ \varepsilon_i^2(t-q) + \beta_{1,i}\ h_i(t-1) + \cdots + \beta_{p,i}\ h_i(t-p)$$

For example, we the GSTAR-ARCH models with three locations, the covariance matrix for $\mathbf{\Sigma}(t) = \mathbf{H}_t$, we write as:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R} \mathbf{D}_t = \begin{pmatrix} \sqrt{h_{11,t}} & 0 & 0 \\ 0 & \sqrt{h_{22,t}} & 0 \\ 0 & 0 & \sqrt{h_{33,t}} \end{pmatrix} \begin{pmatrix} 1 & \rho_{21} & \rho_{31} \\ \rho_{21} & 1 & \rho_{32} \\ \rho_{31} & \rho_{32} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{h_{11,t}} & 0 & 0 \\ 0 & \sqrt{h_{22,t}} & 0 \\ 0 & 0 & \sqrt{h_{33,t}} \end{pmatrix}$$

$$= \begin{pmatrix} h_{11,t} & \rho_{21}\sqrt{h_{11,t}}\sqrt{h_{22,t}} & \rho_{31}\sqrt{h_{11,t}}\sqrt{h_{33,t}} \\ \rho_{21}\sqrt{h_{11,t}}\sqrt{h_{22,t}} & h_{22,t} & \rho_{32}\sqrt{h_{22,t}}\sqrt{h_{33,t}} \\ \rho_{31}\sqrt{h_{11,t}}\sqrt{h_{33,t}} & \rho_{32}\sqrt{h_{22,t}}\sqrt{h_{33,t}} & h_{33,t} \end{pmatrix}$$

with ρ_{21} was correlation between $\varepsilon_{1,t}$, $\varepsilon_{2,t}$ and ρ_{31} was correlation between $\varepsilon_{1,t}$, $\varepsilon_{3,t}$, and then $h_{ii}(t)$ are

$$h_{11,t} = \alpha_0^{(1)} + \alpha_1^{(1)}\varepsilon_{1,t-1}^2$$

$$h_{22,t} = \alpha_0^{(2)} + \alpha_1^{(2)}\varepsilon_{2,t-1}^2$$

$$h_{22,t} = \alpha_0^{(2)} + \alpha_1^{(2)}\varepsilon_{2,t-1}^2\ .$$

To estimates the parameters of (3), we make the models in linear form

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (14)$$

where $\boldsymbol{\varepsilon} \overset{iid}{\sim} N(0, \sigma^2 \mathbf{I}_{TN})$. The parameters estimstion of (4) can be solve by the procedures

1. The first estimation of b_i by OLS.
2. Compute the error by using the bi, then estimate the GARCH parameters by Maximum Likelihood method.

3. Compute the variance
4. Estimate the GSTAR parameter by GLS.

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